On the Regularity Implied by the Hypotheses of Geometry

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Abstract: I describe the history of my successful effort with collaborator Moritz Reintjes to resolve the problem of optimal regularity and Uhlenbeck compactness for affine connections in General Relativity, together with our recent extension of the theory to connections on vector bundles with compact and non-compact Lie groups. We set out to determine whether shock wave solutions of the Einstein equations could always be regularized by coordinate transformation, and finally proved this in the affirmative with an argument based purely on the transformation law for connections. Following a referee comment, we realized our argument extends celebrated theorems of Kazdan-DeTurck and Uhlenbeck from Riemannian to Lorentzian geometry and beyond: Every connection with components $\Gamma \in L^\infty$ and $Riem(\Gamma) \in L^p$, $p > n/2$, can be regularized to optimal regularity $\Gamma \in W^{1,p}$, (one derivative above its curvature), by coordinate transformation–and consequently, sequences of such non-optimal connections are always (locally) compact. Our proof is based on the discovery of the RT-equations, non-invariant elliptic equations for the regularizing transformations, elliptic independent of metric signature, constructed using the Euclidean structure of the coordinate system. By this, Uhlenbeck compactness and optimal regularity are pure logical consequences of the rule which defines how connections transform from one coordinate system to another–the starting hypothesis of geometry.